# Big Bend Community College 

## Intermediate Algebra

MPC 099

## Lab Notebook

## (c) ${ }^{\circ}$

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MPC 099 Module A:
Compound Inequalities and Systems of Equations

## Compound Inequalities - AND




## Compound Inequalities - Tripartite

| Tripartite Inequalities: |  |
| :--- | :--- |
| Be sure when solving to balance on |  |
| Example A $\quad-5 \leq 2 x-17<9$ |  |

Absolute Value Inequalities - Simple


Absolute Value Inequalities - Solving

| To solve we first set up a $\quad\|3 x-5\|>8$ | Example B |
| :--- | :--- |
| Example A |  |

Absolute Value Inequalities - Isolate Absolute


Systems of Equations - Introduction to Substitution


## Systems of Equations - Substitute Expression

| Just as we can replace a variable with a number, we can also replace it with an |  |
| :--- | :--- |
| Whenever we substitute it is important to remember |  |
| Example A |  |
| $\qquad$$y=5 x-3$ <br> $-x-5 y=-11$ | $2 x-6 y=-24$ <br> $x=5 y-22$ |
| Practice A B |  |



| If the variables subtract out to zero then it means either |  |
| :--- | :--- |
| there is | $x+4 y=-7$ <br> $21+3 x=-12 y$ |
| Example A |  |




## Systems of Equations - Multiplying Two Equations



Systems of Equations - Special Cases with Addition

| If the variables subtract out to zero then it means either |  |
| :--- | :--- |
| Example A is |  |
| $\qquad$$2 x-4 y=16$ <br> $3 x-6 y=20$ | Example B |

## Systems of 3 Variables - Simple



| To eliminate a variable, we may have to $\quad$ ___ one or |
| :--- |
| Example A |
| $\qquad$$2 x-2 y-z=8$ <br> $6 x-3 y-3 z=27$ <br> $-3 x-5 y-z=-15$ |

Practice A

| Value Table: |  |
| :--- | :--- |
| The equation always comes from the |  |
| Example A |  |


| Interest Table: |  |
| :--- | :--- |
| The equation always comes from the |  |
| Example A | Example B <br> Sophia invested $\$ 1900$ in one account and $\$ 1500$ <br> in another account that paid $3 \%$ higher interest <br> rate. After one year she had earned $\$ 113$ in <br> interest. At what rates did she invest? <br> another which paid $4 \%$ lower interest. At the end <br> of a year he had earned \$345 in interest. At what <br> rates did he invest? |


| With two variables the equations will come from the ___ and columns. |  |
| :---: | :---: |
| Example A <br> Scott has \$2.15 in his pocket made up of eleven quarters and dimes. How many of each coin does he have? | Example B <br> If 105 people attended a concert and tickets for adults cost $\$ 2.50$ while tickets for children cost $\$ 1.75$ and total receipts for the concert were \$228, how many children and how many adults went to the concert? |
| Practice A | Practice B |


| With two variables the equations will come from the |  |
| :--- | :--- |
| Example A | $\begin{array}{l}\text { Example B } \\ \text { A woman invests } \$ 4600 \text { in two different accounts. } \\ \text { The first paid } 13 \% \text {, the second paid } 12 \% \text { interest. } \\ \text { At the end of the first year she had earned } \$ 586 \text { in } \\ \text { interest. How much was in each account? }\end{array}$ | \(\left.\begin{array}{l}A bank loaned out \$4900 to two different <br>

companies. The first loan had a 4\% interest rate; <br>
the second had a 13 \% interest rate. At the end of <br>
the first year the loan had accrued \$421 in <br>
interest. How much was loaned at each rate?\end{array}\right\}\)


| With two variables the equations will come from the ___ and ___ columns. |  |
| :---: | :---: |
| Example A <br> A chemist needs to create 100 mL of a $38 \%$ acid solution. On hand she has a $20 \%$ acid solution and a $50 \%$ acid solution. How many mL of each should she use? | Example B <br> A coffee distributor needs to mix a coffee blend that normally sells for $\$ 8.90$ per pound with another coffee blend that normally sells for \$11.30 per pound. If the distributor wishes to create 70 pounds of coffee that can sell for \$11.16 per pound, how many pounds of each kind of coffee should the mix? |
| Practice A | Practice B |


| The percentage of acid (or other chemical) in pure acid is <br> The percentage of acid (or other chemical) in water is <br> You need 1425 mL of $10 \%$ alcohol solution. On <br> hand you have a 5\% alcohol mixture and pure <br> alcohol. How much of each should you use? | You need a 60\% methane solution. On hand you <br> have 180 mL of an $85 \%$ methane solution. How <br> much water will you need to add to obtain the <br> desired solution? |
| :--- | :--- |

## MPC 099 Module B: Radicals

| Prime Factorization: |  |
| :--- | :--- | :--- |
| A few prime numbers: |  |
| Example A prime factorization we |  |
| 1350 | Example B |



## Simplify Radicals - Not Perfect Radicals



## Simplify Radicals - With Coefficients

| If there is a coefficient on the radical: $\quad$ Example B |  |
| :--- | :--- |
|  | $-8 \sqrt{600}$ |

Simplify Radicals - Variables


| Simplify: $2 x-5 y+3 x+2 y$ |  |
| :--- | :--- |
| When adding and subtracting radicals we can $2 \sqrt{3}-5 \sqrt{7}+3 \sqrt{3}+2 \sqrt{7}$ |  |
| Example A |  |
| $\qquad-4 \sqrt{6}+2 \sqrt{11}+\sqrt{11}-5 \sqrt{6}$ | $\sqrt[3]{5}+3 \sqrt{5}-8 \sqrt[3]{5}+2 \sqrt{5}$ |


| Before adding radicals together |  |  |
| :--- | :--- | :---: |
| $4 \sqrt{50 x}+5 \sqrt{27}-3 \sqrt{2 x}-2 \sqrt{108}$ | $\sqrt[3]{81 x^{3} y}-3 y \sqrt[3]{32 x^{2}}+x \sqrt[3]{24 y}-\sqrt[3]{500 x^{2} y^{3}}$ |  |
| Example A |  |  |
| Practice A |  |  |


| Product Rule: $a \sqrt[n]{b} \cdot c \sqrt[n]{d}=$ |  |
| :--- | :--- |
| Always be sure your final answer is |  |
| $4 \sqrt{6} \cdot 2 \sqrt{15}$ | $-3 \sqrt[4]{8} \cdot 7 \sqrt[4]{10}$ |

## Add/Subtract/Multiply - Distributing with Radicals

| Recall: $a(b+c)=$ |  |
| :--- | :--- |
| Always be sure your final answer is |  |
| Example A |  |



## Add/Subtract/Multiply - Conjugates





## Rationalize Denominators - Quotient Rule





| What doesn't work: $\frac{1}{2+\sqrt{3}}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Recall: $(2+\sqrt{3})(\quad)$ |  |  |  |
| Multiply by the ___ |  |  |  |
| Example A$\frac{6}{5-\sqrt{3}}$ |  | Example B |  |
|  |  |  | $\frac{3-5 \sqrt{2}}{4+2 \sqrt{2}}$ |


| If we divide the exponent by the index, then $\sqrt[n]{a^{m}}=$ |  |
| :--- | :--- |
| The index is the $\quad$ Write as an exponent: $\sqrt[7]{m^{5}}$ | Example B |
| Example A |  |


| To evaluate a rational exponent | Example B |
| :--- | :--- |
| Example A |  |

## Rational Exponents - Simplify




## Mixed Index - Multiply



Mixed Index - Divide


## MPC 099 Module C: Quadratics

## Complex Numbers - Square Roots of Negatives



## Complex Numbers - Add/Subtract



| Complex Numbers - Multiply |  |
| :---: | :---: |
| $i$ works just like $\qquad$ <br> Remember $i^{2}=$ |  |
| Example A $(-3 i)(6 i)$ | Example B $2 i(5-2 i)$ |
| Example C $(4-3 i)(2-5 i)$ | Example D $(3+2 i)^{2}$ |
| Practice A | Practice B |
| Practice C | Practice D |



## Complex Numbers - Rationalize Binomials





## Equations with Radicals - Isolate Radical

| Before we can clear a radical it must first be |  |
| :--- | :--- | :--- |
| Example A |  |
| $\qquad 4+2 \sqrt{2 x-1}=2 x$ |  |



| Practice A Practice B |
| :--- | :--- | :--- |


| The opposite of taking an exponent is to do a |  |
| :--- | :--- |
| If $x^{3}=8$, then $x=$ |  |
| Example A |  |
|  |  |


| Consider: $\left(5^{2}\right)=\quad$ and $(-5)^{2}=$ |  |
| :--- | :--- |
| When we clear an even root we have |  |
| Example A $\quad(5 x-1)^{2}=49$ |  |

Equations with Exponents - Isolate Exponent


| To multiply to one: $\frac{a}{b} *(-)=1$ |  |
| :--- | :--- |
| We clear a rational exponent by using a |  |
| Recall: $a^{m / n}=$ |  |
| Recall: Check answer if $\quad(3 x-6)^{2 / 3}=64$ | $(5 x+1)^{5 / 4}=32$ |
| Example A $\quad$ Example B |  |


| $a^{2}+2 a b+b^{2}$ is easily factored to |  |
| :---: | :---: |
| To make $x^{2}+b x+c$ a perfect square, $c=$ |  |
| Example A <br> Find $c$ and factor the perfect square $x^{2}+10 x+c$ | Example B <br> Find $c$ and factor the perfect square $x^{2}-7 x+c$ |
| Example C <br> Find $c$ and factor the perfect square $x^{2}-\frac{3}{7} x+c$ | Example D <br> Find $c$ and factor the perfect square $x^{2}+\frac{6}{5} x+c$ |
| Practice A | Practice B |
| Practice C | Practice D |



| If we can't simplify the $\ldots$ Example B whe wan. |  |
| :--- | :--- |
| Example A $2 x^{2}-8 x-3=0$ | $5 x^{2}-3 x+2=0$ |

Quadratic Formula - Finding the Formula

Solve by completing the square:

$$
a x^{2}+b x+c=0
$$

## Quadratic Formula - Using the Formula

| If $a x^{2}+b x+c=0$ then $x=$ |  |
| :--- | :--- |
| Example A | Example B |


| Before using the quadratic formula, the equation must equal |  |
| :--- | :--- |
| Example A |  |
| $2 x^{2}=15-7 x$ | $3 x^{2}+5 x+2=7$ |

Quadratic Formula $-b=0$

| If a term is missing, we use_____ in the quadratic formula. |  |
| :--- | :--- |
| Example A |  |


| Area of a rectangle: |  |
| :--- | :--- |
| To help visualize the rectangle, |  |
| Example A <br> The length of a rectangle is 2 ft longer than the <br> width. The area of the rectangle is $48 \mathrm{ft}^{2}$. What <br> are the dimensions of the rectangle? | The area of a rectangle is $72 \mathrm{~cm}^{2}$. If the length is 6 <br> cm more than the width, what are the dimensions <br> of the rectangle? |

## Rectangles - Perimeter

| Perimeter of a Rectangle: |  |
| :--- | :--- |
| Tip: Solve the _equation for a variable. |  |
| The area of a rectangle is $54 \mathrm{~m}^{2}$. If the perimeter is <br> 30 meters, what are the dimensions of the <br> rectangle? | Example B <br> The perimeter of a rectangle is 22 inches. If the <br> area of the same rectangle is $24 \mathrm{in}^{2}$, what are the <br> dimensions? |


| We may have to draw ___ rectangles. |  |
| :---: | :---: |
| Multiply/Add to the $\qquad$ to get the big rectangle. |  |
| Divide/Subtract to the ___ to get the small rectangle. |  |
| Example A <br> Each side of a square is increased 6 inches. When this happens, the area is multiplied by 16 . How many inches in the side of the original square? | Example B <br> The length of a rectangle is 9 feet longer than it is wide. If each side is increased 9 feet, then the area is multiplied by 3 . What are the dimensions of the original rectangle? |
| Practice A | Practice B |






## MPC 099 Module D: <br> Rational Equations

## Rational Equations - Clear Denominator

| Recall: $\quad \frac{3}{4} x-\frac{1}{2}=\frac{5}{6}$ |  |
| :---: | :---: |
| Clear fractions by multiplying ___ by ___ |  |
| Example A $\frac{5}{x}=\frac{3}{7 x}-4$ | Example B $\frac{4}{x+5}+x=\frac{-2}{x+5}$ |
| Practice A | Practice B |



## Rational Equations - Extraneous Solutions



## Work Problems - One Unknown Time

| Adam does a job in 4 hours. Each hour he does ____ of the job. |  |
| :---: | :---: |
| Betty does a job in 12 hours. Each hour he does ___ of the job. |  |
| Together, each hour they do $\qquad$ of the job. |  |
| This means it takes them, working together, $\qquad$ hours to do the entire job. |  |
| Work Equation: Use | __! |
| Example A <br> Catherine can paint a house in 15 hours. Dan can paint it in 30 hours. How long will it take them working together? | Example B <br> Even can clean a room in 3 hours. If his sister Faith helps, it takes them $2 \frac{2}{5}$ hours. How long will it take Faith working alone? |
| Practice A | Practice B |




| Revenue Table: |  |
| :--- | :--- |
| To solve: Divide by |  |
| Example A <br> A group of college students bought a couch for <br> \$80. However, five of them failed to pay their <br> share so the others had to each pay $\$ 8$ more. How <br> many students were in the original group? | A merchant bought several pieces of silk for $\$ 70$. <br> He sold all but two of them at a profit of $\$ 4$ per <br> piece. His total profit was $\$ 18$. How many pieces <br> did he originally purchase? |


| Practice A Practice B |
| :--- | :--- | :--- |


| Distance Table: |  |
| :--- | :--- |
| To solve: Divide by |  |
| Example A man rode his bike to a park 60 miles away. On <br> A me return trip he went 2 mph slower which made <br> the trip take 1 hour longer. How fast did he ride to <br> the park? | After driving through a construction zone for 45 <br> miles, a woman realized that if she had driven just <br> 6 mph faster she would have arrived 2 hours <br> sooner. How fast did she drive? |


| Practice A Practice B |
| :--- | :--- | :--- |


| Downwind/stream: <br> Upwind/stream: <br> Example A <br> Zoe rows a boat downstream for 80 miles. The <br> return trip upstream took 12 hours longer. If the <br> current flows at 3 mph, how fast does Zoe row in <br> still water? <br> Example B <br> Darius flies a plane against a headwind for 5084 <br> miles. The return trip with the wind took 20 hours <br> less time. If the wind speed is 10 mph , how fast <br> does Darius fly the plane when there is no wind? <br> (practice problems on the next page) |  |
| :--- | :--- |


| Practice A Practice B |
| :--- | :--- |


| To help visualize the frame <br> Remember the frame is on the $\qquad$ and $\qquad$ , also the $\qquad$ and $\qquad$ |  |
| :---: | :---: |
| Example A <br> A picture measures 10 inches by 7 inches is placed in a frame of uniform width. If the area of the frame and picture together is 208 square inches, what is the width of the frame? | Example B <br> An 8 inch by 12 inch drawing has a frame of uniform width around it. The area of the frame is equal to the area of the picture. What is the width of the frame? |
| Practice A | Practice B |

Clearly identify the area of the $\qquad$ and $\qquad$ rectangles!

Be careful with $\qquad$ is it talking about the $\qquad$ or $\qquad$ ?
Example A
A man mows his 40 ft by 50 ft rectangular lawn in a spiral pattern starting from the outside edge. By noon he is $90 \%$ done. How wide of a strip has he cut around the outside edge?

Example B
A farmer has a 50 ft by 25 ft rectangular field that he wants to increase by $68 \%$ by cultivating a strip of uniform width around the current field. How wide of a strip should he cultivate?

Practice B

## MPC 099 Module E: Functions

## Functions - Definition and Vertical Line Test




## Functions - Function Notation



## Functions - Evaluate at Expressions



## Algebra of Functions - Add/Subtract/Multiply/Divide



| Composition of Functions:$(f \circ g)(x)=f(g(x))$ |  |
| :---: | :---: |
| Example A $(g \circ f)(7)=$ | Example B $(r \circ p)(x)=$ |
| Practice A | Practice B |

## Inverse Functions - Showing Functions are Inverses




Inverse Functions - Graph the Inverse



## Exponential Equations - Common Base



## Exponential Functions - Binomial Exponents

| When multiplying exponents we may have to |  |
| :--- | :--- | :--- |
| Example A |  |
| $\qquad 8^{2 x-4}=16^{x+3}$ |  |


| Compound Interest: |  |
| :---: | :---: |
| $n$ compounds per year: $A=P\left(1+\frac{r}{n}\right)^{n t}$ |  |
| $A=$ |  |
| $P=$ |  |
| $r=$ |  |
| $n=$ |  |
| $t=$ |  |
| Example A | Example B |
| Suppose you invest $\$ 13000$ in an account that pays $8 \%$ interest compounded monthly. How much would be in the account after 9 years? | You loan out \$800 to you friend at 3\% interest compounded quarterly. Your friend pays you back after five years. What does he owe you? |
| Practice A | Practice B |


| Evaluate__first |  |
| :--- | :--- |
| Example A <br> How much money would have to be invested at <br> $6 \%$ interest compounded weekly to be worth <br> $\$ 1500$ at the end of 15 years?What principle would amount to $\$ 800$ if invested <br> for 10 years at $12 \%$ interest compounded semi- <br> annually? |  |
| Practice A |  |


| Continuous Interest: |  |
| :--- | :--- |
| $A=P e^{r t}$ |  |
| $P=$ |  |
| $e=$ |  |
| $r=$ |  |
| $t=$ |  |
| Example A |  |
| An investment of \$25000 is at an interest rate of | What is the balance at the end of 10 years on an |
| $11.5 \%$ compounded continuously. What is the <br> balance after 20 years? <br>  | investment of $\$ 13000$ at 4\% compounded <br> continuously? |

Compound Interest - Finding Principle with Continuous Interest

| Evaluate___first |  |
| :--- | :--- |
| Example A <br> To pay an \$1100 vacation in 10 years, how much <br> money should the Franklins invest at 9\% interest <br> compounded continuously? | How much should you invest at $12 \%$ continuous <br> interest for 100 years in order to have \$1,000,000? |


| Logarithm: <br> $b^{x}=a$ can be written as $\qquad$ |  |
| :---: | :---: |
| Example A <br> Write each as a logarithm: $m^{2}=25$ $5^{x}=125$ | Example B <br> Write each as an exponent: $\log _{x} 64=2$ $\log _{5} x=m$ |
| Practice A | Practice B |



| To solve a logarithmic equation: |  |
| :--- | :--- |
| Example A $\log _{x} 8=3$ |  |

